

Complete Set of Splitting Functions Relevant in the Evolution of Nucleonic Helicity Distributions

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In this work, we show how the complete set of splitting functions relevant for the evolution of various distribution functions describing nucleonic helicity structure can be obtained in the light front Hamiltonian perturbation theory using completely fixed light front gauge, $A^+ = 0$.

I. INTRODUCTION

How the total helicity of a nucleon is distributed among its constituents is an active area of research at present. In a previous work [1]¹, we have shown that in a *gauge fixed* theory (with $A^+ = 0$ gauge) total helicity operator can be satisfactorily defined and separated into orbital and intrinsic parts for quarks and gluons and defined the structure functions containing information regarding the orbital helicity distribution of the constituents. There we also employed our newly proposed methods of calculating dressed parton structure functions in light front Hamiltonian framework [2] and obtained some of the splitting functions which are relevant for the Q^2 -evolution of these structure functions and the corresponding anomalous dimensions, which were identical with the recent results [3] using different techniques (but with the same $A^+ = 0$ gauge). In this work, we shall use our gauge fixed formulation and find out the complete set of splitting functions relevant in the evolution of various helicity distributions to complete the picture.

In our previous calculation of the various structure functions in paper I, we used the dressed quark and gluon targets which are eigenstates of $P \equiv (P^+, P^1, P^2)$ and S^3 . It was implicitly assumed there that the total $P^\perp \equiv (P^1, P^2)$ for the targets to be zero, which one could always do by appropriately orienting the 3-axis. This in turn forced the orbital motion of the dressed parton associated with its centre of mass motion to vanish. Consequently, our calculation produced the restricted number of splitting functions which are important for the QCD evolution of the assumed state. To obtain the complete set of relevant splitting functions and the corresponding anomalous dimensions [3] which, for example, takes into account how the orbital motion evolves under QCD interaction, one has to consider a target state where orbital motion of the constituents is present even in absence of QCD interaction. This is nothing but due to the centre of mass motion of the dressed parton. Note that this centre of mass motion has nothing to do with that of a composite system such as meson. For the calculation involving a meson target, it will be replaced by the internal motion of the concerned parton. We shall introduce the relevant target state for our calculation in Sec.II,

¹Henceforth referred, frequently in this work to avoid repetition, as paper I.

where we have also introduced the multi-parton wave-functions relevant for our calculations. In Secs.III-IV we present our calculations and then conclude in Sec.V.

II. THE TARGET STATE AND MULTI-PARTON WAVE-FUNCTIONS

Required state for the dressed parton can be obtained by taking a suitable superposition of states where all the individual states carry definite longitudinal and transverse momenta (P) as well as the intrinsic helicity (σ) and then projecting out those states which carry same j^3 by introducing a Kronecker-delta function. We consider the following as our target state

$$|S\rangle = \sum_{\sigma} \sum_m e^{im\phi} \Phi_{\text{cm}}^{j^3}(P^+, |P^\perp|, m) \delta_{m, j^3 - \sigma} |P, \sigma\rangle, \quad (1)$$

where ϕ parametrizes the state such that $(P^1, P^2) = |P^\perp|(\cos \phi, \sin \phi)$. Note that we have written the ϕ -dependence separately as a sum over m , the eigenvalue of L^3 , so that Φ_{cm} is independent of ϕ . The dressed parton state $|P, \sigma\rangle$ in the RHS of eq.(1) can be written in the Fock-basis using multi-parton wave-functions which are independent of momenta P [4]. For a dressed quark, it is given by

$$|P^+, P_\perp, \sigma\rangle = \sqrt{\mathcal{N}_q} \left\{ b^\dagger(k, \sigma) |0\rangle + \sum_{\sigma_1 \lambda} \int \frac{dk_1^+ d^2 k_{\perp 1}}{\sqrt{2(2\pi)^3 k_1^+}} \frac{dk_2^+ d^2 k_{\perp 2}}{\sqrt{2(2\pi)^3 k_2^+}} \sqrt{2(2\pi)^3 P^+} \delta^3(P - k_1 - k_2) \right. \\ \left. \times \Phi_{\sigma_1 \lambda}^\sigma(P; k_1, k_2) b^\dagger(k_1, \sigma_1) a^\dagger(k_2, \lambda) |0\rangle + \dots \right\}, \quad (2)$$

where the normalization constant \mathcal{N}_q is determined perturbatively from the normalization condition,

$$\langle P'^+, P'_\perp, \sigma' | P^+, P_\perp, \sigma \rangle = 2(2\pi)^3 P^+ \delta_{\sigma, \sigma'} \delta(P^+ - P'^+) \delta^2(P_\perp - P'_\perp) \dots \quad (3)$$

Similarly for a dressed gluon

$$|P^+, P_\perp, \lambda\rangle = \sqrt{\mathcal{N}_g} \left\{ a^\dagger(k, \sigma) |0\rangle + \sum_{\sigma_1 \sigma_2} \int \frac{dk_1^+ d^2 k_{\perp 1}}{\sqrt{2(2\pi)^3 k_1^+}} \frac{dk_2^+ d^2 k_{\perp 2}}{\sqrt{2(2\pi)^3 k_2^+}} \sqrt{2(2\pi)^3 P^+} \delta^3(P - k_1 - k_2) \right. \\ \times \Phi_{\sigma_1 \sigma_2}^\lambda(P; k_1, k_2) b^\dagger(k_1, \sigma_1) d^\dagger(k_2, \sigma_2) |0\rangle \\ \left. + \frac{1}{2} \sum_{\lambda_1 \lambda_2} \int \frac{dk_1^+ d^2 k_{\perp 1}}{\sqrt{2(2\pi)^3 k_1^+}} \frac{dk_2^+ d^2 k_{\perp 2}}{\sqrt{2(2\pi)^3 k_2^+}} \sqrt{2(2\pi)^3 P^+} \delta^3(P - k_1 - k_2) \right. \\ \left. \times \Phi_{\lambda_1 \lambda_2}^\lambda(P; k_1, k_2) a^\dagger(k_1, \lambda_1) a^\dagger(k_2, \lambda_2) |0\rangle + \dots \right\}. \quad (4)$$

We introduce the boost invariant amplitudes $\psi_{\sigma_1, \lambda_2}^\sigma(x, \kappa^\perp)$ and so on by $\Phi_{\lambda_1 \lambda_2}^\sigma(P; k_1, k_2) = \frac{1}{\sqrt{P^+}} \psi_{\sigma_1 \lambda_2}^\sigma(x, \kappa^\perp)$ and so on. Here the relative momenta (x, κ^\perp) are defined as

$$k_1^+ = xP^+, \quad k_1^i = \kappa^i + xP^i \quad \text{and} \quad k_2^+ = (1-x)P^+, \quad k_2^i = -\kappa^i + (1-x)P^i. \quad (5)$$

From the light-front QCD Hamiltonian, to the lowest order in perturbation theory, we have [2],

$$\psi_{\sigma_1 \lambda_2}^{\sigma | q \bar{q}}(x, \kappa_{\perp}) = -\frac{g}{\sqrt{2(2\pi)^3}} T^a \frac{x(1-x)}{\kappa_{\perp}^2} \chi_{\sigma_1}^{\dagger} \left\{ 2 \frac{\kappa_{\perp}^i}{1-x} + \frac{1}{x} (\sigma_{\perp} \cdot \kappa_{\perp}) \sigma^i \right\} \chi_{\sigma} \varepsilon_{\lambda_2}^{i*}, \quad (6)$$

$$\psi_{\sigma_1 \sigma_2}^{\lambda | q \bar{q}}(x, \kappa_{\perp}) = -\frac{g}{\sqrt{2(2\pi)^3}} T^a \frac{x(1-x)}{\kappa_{\perp}^2} \chi_{\sigma_1}^{\dagger} \left\{ \frac{(\sigma_{\perp} \cdot \kappa_{\perp})}{x} \sigma_{\perp}^i - \sigma_{\perp}^i \frac{(\sigma_{\perp} \cdot \kappa_{\perp})}{1-x} \right\} \chi_{-\sigma_2} \varepsilon_{\lambda}^i, \quad (7)$$

$$\psi_{\lambda_1 \lambda_2}^{\lambda | g g}(x, \kappa_{\perp}) = -\frac{2igf^{abc}}{\sqrt{2(2\pi)^3}} \frac{\sqrt{x(1-x)}}{\kappa^2} \left\{ -\kappa^i \delta_{jl} + \frac{\kappa^j}{x} \delta_{il} + \frac{\kappa^l}{1-x} \delta_{ij} \right\} \varepsilon_{\lambda_1}^{j*} \varepsilon_{\lambda_2}^{l*} \varepsilon_{\lambda}^i. \quad (8)$$

Here we have already taken target mass and bare quark mass to be zero, $M = m = 0$, since nonzero masses in the above wave-functions are going to produce higher twist effects. Our next task is to calculate the structure functions with $|S\rangle$ being the dressed quark target [*i.e.*, eq.(1) and eq.(2) combined] and dressed gluon target [*i.e.*, eq.(1) and eq.(4) combined].

III. DRESSED QUARK STRUCTURE FUNCTIONS

For the dressed quark eq.(1) can be written as

$$|S\rangle = e^{im_1\phi} \Phi_{\text{cm}}(m_1) |P, +\frac{1}{2}\rangle + e^{im_2\phi} \Phi_{\text{cm}}(m_2) |P, -\frac{1}{2}\rangle, \quad (9)$$

where $m_1 = j^3 - \frac{1}{2}$ and $m_2 = j^3 + \frac{1}{2}$. For such a state, in the zeroth order perturbation theory, *i.e.*, if the two-particle wave-function in eq.(2) is zero, $\Delta q(x, Q^2)$ and $\Delta q_L(x, Q^2)$ come out to be proportional to δ -functions and are given by²

$$\begin{aligned} \Delta q(x, Q^2) &= \frac{\mathcal{N}_q}{2} \left\{ |\Phi_{\text{cm}}(m_1)|^2 - |\Phi_{\text{cm}}(m_2)|^2 \right\} \delta(1-x) \equiv \frac{1}{2} \Delta \Sigma \delta(1-x), \\ \Delta q_L(x, Q^2) &= \mathcal{N}_q \left\{ m_1 |\Phi_{\text{cm}}(m_1)|^2 + m_2 |\Phi_{\text{cm}}(m_2)|^2 \right\} \delta(1-x) \equiv L_q \delta(1-x), \end{aligned} \quad (10)$$

while $\Delta g(x, Q^2)$ and $\Delta g_L(x, Q^2)$ are zero. Of course, $\mathcal{N}_q = 1$ has been used above, since calculations are performed only in zeroth order in the coupling.

Now, the structure functions can be calculated to the first order in light-front perturbation theory by truncating the dressed quark state in eq.(2) at the two particle level and using the wave-function in eq.(6). Straight-forward calculations give

$$\Delta q(x, Q^2) = \frac{\mathcal{N}_q}{2} \Delta \Sigma \left\{ \delta(1-x) + \frac{\alpha_s}{2\pi} C_f \ln \frac{\Lambda^2}{\mu^2} \frac{1+x^2}{1-x} \right\}, \quad (11)$$

where Λ and μ are the upper and lower momentum cutoff. Note that it is same as we obtained in paper I except an extra weight factor $\Delta \Sigma$ coming from the fact that the target state now is a particular superposition. Similarly,

$$\Delta g(1-x, Q^2) = \mathcal{N}_q \Delta \Sigma \frac{\alpha_s}{2\pi} C_f \ln \frac{\Lambda^2}{\mu^2} (1+x). \quad (12)$$

² See paper I for the definitions of various structure functions.

On the other hand, we get extra terms while calculating $\Delta q_L(x, Q^2)$ and $\Delta g_L(1-x, Q^2)$ as given below.

$$\begin{aligned}\Delta q_L(x, Q^2) = & -\mathcal{N}_q \Delta\Sigma \frac{\alpha_s}{2\pi} C_f \ln \frac{\Lambda^2}{\mu^2} (1-x)(1+x) \\ & + \mathcal{N}_q L_q \left\{ \delta(1-x) + \frac{\alpha_s}{2\pi} C_f \ln \frac{\Lambda^2}{\mu^2} \frac{x^3+x}{1-x} \right\}\end{aligned}\quad (13)$$

$$\begin{aligned}\Delta g_L(1-x, Q^2) = & -\mathcal{N}_q \Delta\Sigma \frac{\alpha_s}{2\pi} C_f \ln \frac{\Lambda^2}{\mu^2} x(1+x) \\ & + \mathcal{N}_q L_q \frac{\alpha_s}{2\pi} C_f \ln \frac{\Lambda^2}{\mu^2} (1+x^2)\end{aligned}\quad (14)$$

The extra terms in eq.(13) and eq.(14) are coming due to the fact that we have used [1]

$$\begin{aligned}\hat{L}_q^3 = & -i \left[(1-x) \frac{\partial}{\partial\theta} + x \frac{\partial}{\partial\phi} \right] \\ \hat{L}_g^3 = & -i \left[x \frac{\partial}{\partial\theta} + (1-x) \frac{\partial}{\partial\phi} \right]\end{aligned}\quad (15)$$

where θ is defined by $(\kappa^1, \kappa^2) \equiv |\kappa^\perp|(\cos\theta, \sin\theta)$. In our previous calculation $\frac{\partial}{\partial\phi}$ did not contribute for we assumed $P^\perp = 0$. Also notice that the other terms in $\hat{L}_{q,g}^3$ (as given in paper I) have no contribution for they produced odd integral of κ^\perp over symmetric limit.

Now, the normalization constant \mathcal{N}_q to the order α_s can be calculated from eq.(3) and is given by [2]

$$\begin{aligned}\mathcal{N}_q = & \left\{ 1 - \frac{\alpha_s}{2\pi} C_f \ln \frac{\Lambda^2}{\mu^2} \int dx \frac{1+x^2}{1-x} \right\}, \\ = & 1 - C_f \frac{\alpha_s}{2\pi} \ln \frac{\Lambda^2}{\mu^2} \left[\int dx \frac{2}{1-x} - \frac{3}{2} \right].\end{aligned}\quad (16)$$

Putting \mathcal{N}_q back into eqs.(11-14) and keeping terms only to the order α_s , we have

$$\begin{aligned}\Delta q(x, Q^2) = & \frac{1}{2} \Delta\Sigma \left\{ \delta(1-x) + \frac{\alpha_s}{2\pi} \ln \frac{\Lambda^2}{\mu^2} P_{SS(qq)}(x) \right\}, \\ \Delta g(1-x, Q^2) = & \Delta\Sigma \frac{\alpha_s}{2\pi} \ln \frac{\Lambda^2}{\mu^2} P_{SS(gq)}(1-x), \\ \Delta q_L(x, Q^2) = & \Delta\Sigma \frac{\alpha_s}{2\pi} \ln \frac{\Lambda^2}{\mu^2} P_{LS(gq)}(x) \\ & + L_q \left\{ \delta(1-x) + \frac{\alpha_s}{2\pi} \ln \frac{\Lambda^2}{\mu^2} P_{LL(qq)}(x) \right\}, \\ \Delta g_L(1-x, Q^2) = & \Delta\Sigma \frac{\alpha_s}{2\pi} \ln \frac{\Lambda^2}{\mu^2} P_{LS(gq)}(1-x) \\ & + L_q \frac{\alpha_s}{2\pi} \ln \frac{\Lambda^2}{\mu^2} P_{LL(gq)}(1-x),\end{aligned}\quad (17)$$

where we have defined the various splitting functions as follows. (Notice that the argument of gluon distribution functions is $(1-x)$ which is the momentum fraction carried by the gluon here.)

$$\begin{aligned}
P_{SS(qq)}(x) &= C_f \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x) \right], \\
P_{SS(gq)}(1-x) &= C_f (1+x) \\
P_{LS(qq)}(x) &= -C_f (1-x^2) \\
P_{LS(gq)}(1-x) &= -C_f x(1+x) \\
P_{LL(qq)}(x) &= C_f \left[\frac{x^3+x}{(1-x)_+} + \frac{3}{2}\delta(1-x) \right], \\
P_{LL(gq)}(1-x) &= C_f (1+x^2)
\end{aligned} \tag{18}$$

Also, since there are no contributions coming from L_q in $\Delta q(x, Q^2)$ and $\Delta g(x, Q^2)$, we have,

$$P_{SL(qq)}(x) = P_{SL(gq)}(x) = 0. \tag{19}$$

Last two splitting functions in eq.(18) and that in eq.(19) are new compared to those in paper I.

IV. DRESSED GLUON STRUCTURE FUNCTIONS

Now we repeat the calculations given in the previous section by replacing the target state with a dressed gluon. For the dressed gluon eq.(1) can be written as

$$|S\rangle = e^{im_1\phi} \Phi_{\text{cm}}(m_1) |P, +1\rangle + e^{im_2\phi} \Phi_{\text{cm}}(m_2) |P, -1\rangle, \tag{20}$$

where $m_1 = j^3 - 1$ and $m_2 = j^3 + 1$. For such a state, in contrast to the quark case, $\Delta g(x, Q^2)$ and $\Delta g_L(x, Q^2)$ are proportional to the δ -function in the zeroth order perturbation theory while $\Delta q(x, Q^2)$ and $\Delta q_L(x, Q^2)$ are zero.

$$\begin{aligned}
\Delta g(x, Q^2) &= \mathcal{N}_g \left\{ |\Phi_{\text{cm}}(m_1)|^2 - |\Phi_{\text{cm}}(m_2)|^2 \right\} \delta(1-x) \equiv \Delta g \delta(1-x), \\
\Delta g_L(x, Q^2) &= \mathcal{N}_g \left\{ m_1 |\Phi_{\text{cm}}(m_1)|^2 + m_2 |\Phi_{\text{cm}}(m_2)|^2 \right\} \delta(1-x) \equiv L_g \delta(1-x),
\end{aligned} \tag{21}$$

Now, with the dressed gluon state truncated at the two-particle level as given in eq.(4), we can calculate all the structure functions to first order in α_s and the results are as follows

$$\begin{aligned}
\Delta q(x, Q^2) &= \Delta g \frac{\alpha_s}{2\pi} \ln \frac{\Lambda^2}{\mu^2} P_{SS(qg)}(x) \\
\Delta q_L(x, Q^2) &= \Delta g \frac{\alpha_s}{2\pi} \ln \frac{\Lambda^2}{\mu^2} 2P_{LS(qg)}(x) + L_g \frac{\alpha_s}{2\pi} \ln \frac{\Lambda^2}{\mu^2} 2P_{LL(qg)}(x), \\
\Delta g(x, Q^2) &= \Delta g \left\{ \delta(1-x) + \frac{\alpha_s}{2\pi} \ln \frac{\Lambda^2}{\mu^2} P_{SS(gg)}(x) \right\}, \\
\Delta g_L(x, Q^2) &= \Delta g \frac{\alpha_s}{2\pi} \ln \frac{\Lambda^2}{\mu^2} P_{LS(gg)}(x) \\
&\quad + L_g \left\{ \delta(1-x) + \frac{\alpha_s}{2\pi} \ln \frac{\Lambda^2}{\mu^2} P_{LL(gg)}(x) \right\}
\end{aligned} \tag{22}$$

where, like the quark case, we have defined the splitting functions as follows.

$$\begin{aligned}
P_{SS(qg)}(x) &= \frac{1}{2} [x^2 - (1-x)^2], \\
P_{SS(gg)}(x) &= N \left\{ (1+x^4) \left[\frac{1}{x} + \frac{1}{(1-x)_+} \right] - \frac{(1-x)^3}{x} \right\} + \delta(1-x) \left(\frac{11}{6}N - \frac{1}{3} \right), \\
P_{LS(qg)}(x) &= \frac{1}{2} [x^2 + (1-x)^2] (1-x), \\
P_{LS(gg)}(x) &= 2N (x-1)(x^2 - x + 2), \\
P_{LL(qg)}(x) &= \frac{1}{2} x [x^2 + (1-x)^2], \\
P_{LL(gg)}(x) &= 2N x \left\{ \frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right\} + \delta(1-x) \left(\frac{11}{6}N - \frac{1}{3} \right), \tag{23}
\end{aligned}$$

and

$$P_{SL(qg)}(x) = P_{SL(gg)}(x) = 0. \tag{24}$$

Here, in the above calculation, we have used the normalization constant \mathcal{N}_g as [2]

$$\begin{aligned}
\mathcal{N}_g &= 1 - \frac{\alpha_s}{2\pi} \ln \frac{\Lambda^2}{\mu^2} \int dx \left[2Nx \left\{ \frac{x}{1-x} + \frac{1-x}{x} + x(1-x) \right\} + \frac{1}{2} [x^2 + (1-x)^2] \right], \\
&= 1 - \frac{\alpha_s}{2\pi} \ln \frac{\Lambda^2}{\mu^2} \int dy \frac{2N}{1-y} + \left(\frac{11}{6}N - \frac{1}{3} \right). \tag{25}
\end{aligned}$$

Last two splitting functions in eq.(23) and that in eq.(24) are new compared to those in paper I. Also, to the best of our knowledge, spin-dependent Altarelli-Parisi splitting functions denoted here as $P_{SS(qg)}$ and $P_{SS(gg)}$ are explicitly derived using light-front Hamiltonian perturbation theory for the first time here.

Corresponding anomalous dimensions are defined as

$$A_{AB(ab)}^n = \int dx x^{n-1} P_{AB(ab)}(x). \tag{26}$$

Using this definition the results given in Ref. [3] for the anomalous dimensions are exactly reproduced and it is needless to rewrite them here. Note that for these dressed parton states, expectation value of J^3 to this order in perturbation theory is independent of α_s , as it should be due to kinematical nature of the helicity operator,

$$\begin{aligned}
\frac{\langle S | J^3 | S \rangle_q}{2(2\pi)^3 P^+ \delta^3(0)} &= \frac{1}{2} \Delta \Sigma + L_q, \\
\frac{\langle S | J^3 | S \rangle_g}{2(2\pi)^3 P^+ \delta^3(0)} &= \Delta g + L_g. \tag{27}
\end{aligned}$$

Eqs.(27) explicitly verify the helicity sum rule for the two different states considered here and show the consistency of our calculation.

V. CONCLUSION

In this work, we have presented the calculations of the complete set of splitting functions relevant in the evolution of the various helicity distribution functions defined in the gauge $A^+ = 0$. This work in association with our previous work in paper I gives a satisfactory description of various parts of helicity distributions and their evolutions in the *gauge fixed light front Hamiltonian formulation* and in conformity with the work presented in Ref. [3].

It should be noted that our formulation and results are obtained in a gauge fixed theory (same is true for the calculation in Ref. [3]). At present, we have very little idea in which processes the orbital helicity distributions that we have defined can be measured experimentally. It needs to be admitted that in absence of the experimental support and/or our results are reproduced by some other means, the *gauge fixed* formulation is little unsettled. On the other hand, gauge invariance in separating the helicity operator into intrinsic and orbital parts for quarks and gluons is a long standing issue. Lot of work has been done in the last few years towards defining such operators showing full respect to the gauge invariance [5]. Also, there are attempts to define operators compatible to the residual gauge invariance in light-front gauge [6]. In spite of all these works, the issue still seems to be a little unsettled [7] as well. In view of this dilemma, it needs thorough investigation to settle down the question of gauge invariance and/or find out processes sensitive to OAM that can be measured in the experiment. We hope to undertake such investigations in future.

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APPENDIX A:

In this appendix, we provide the necessary details of calculation in arriving at the expression for $\Delta q_L(x, Q^2)$ given eq.(13) starting from its definition as an example. All the other splitting functions can be obtained in a similar manner. We have (see paper I),

$$\Delta q_L(x, Q^2) = \frac{1}{4\pi P^+} \int d\eta e^{-i\eta x} \langle S | [\bar{\psi}(\xi^-) \gamma^+ i(x^1 \partial^2 - x^2 \partial^1) \psi(0) + h.c.] | S \rangle \quad (A1)$$

with $\eta = \frac{1}{2}P^+\xi^-$. Note that the operator involved here is a number conserving operator, so that the non-vanishing contributions will only be diagonal in particle number. Let us first consider the contribution coming from the first term in $|S\rangle$ given in eq.(9). Using the Fourier expansion for the dynamical fermionic field ψ^+ and the standard commutation relation between creation and annihilation operators (see paper I), we get two non-vanishing contributions. First one coming from the one particle sector and is simply given by

$$\begin{aligned} \Delta q_L(1) &= \mathcal{N}_q |\Phi_{\text{cm}}(m_1)|^2 e^{-im_1\phi} (-i \frac{\partial}{\partial \phi}) e^{im_1\phi} \delta(1-x) \\ &= \mathcal{N}_q m_1 |\Phi_{\text{cm}}(m_1)|^2 \delta(1-x). \end{aligned} \quad (A2)$$

The second one coming from the two particle sector is given by

$$\begin{aligned}
\Delta q_L(2) &= \mathcal{N}_q |\Phi_{\text{cm}}(m_1)|^2 \sum_{\sigma_1, \lambda_2} \int d^2 \kappa^\perp e^{-im_1 \phi} \psi_{\sigma_1 \lambda_2}^\dagger{}^*(x, \kappa^\perp) \\
&\quad (-i) \left[(1-x) \frac{\partial}{\partial \theta} + x \frac{\partial}{\partial \phi} \right] \psi_{\sigma_1 \lambda_2}^\dagger(x, \kappa^\perp) e^{im_1 \phi} \\
&= \mathcal{N}_q |\Phi_{\text{cm}}(m_1)|^2 \sum_{\sigma_1, \lambda_2} \int d^2 \kappa^\perp (1-x) \psi_{\sigma_1 \lambda_2}^\dagger{}^*(x, \kappa^\perp) \left(-i \frac{\partial}{\partial \theta} \right) \psi_{\sigma_1 \lambda_2}^\dagger(x, \kappa^\perp) \\
&\quad + \mathcal{N}_q m_1 |\Phi_{\text{cm}}(m_1)|^2 \sum_{\sigma_1, \lambda_2} \int d^2 \kappa^\perp x \psi_{\sigma_1 \lambda_2}^\dagger{}^*(x, \kappa^\perp) \psi_{\sigma_1 \lambda_2}^\dagger(x, \kappa^\perp) \quad (\text{A3})
\end{aligned}$$

Here and in the following \uparrow implies helicity $+\frac{1}{2}$ for quarks and $+1$ for gluons and so on. Using $\chi_{\frac{1}{2}} = (1, 0)$, $\chi_{-\frac{1}{2}} = (0, 1)$ and $\varepsilon_{\pm 1} = \frac{1}{\sqrt{2}}(1, \pm i)$ in eq.(6), various components of the two-particle wave-function can be simplified and are given by

$$\begin{aligned}
\psi_{\uparrow\uparrow}^\dagger(x, \kappa^\perp) &= -\frac{gT^a}{\sqrt{2(2\pi)^3}} \frac{\sqrt{2}e^{-i\theta}}{|\kappa^\perp|\sqrt{1-x}}, & \psi_{\uparrow\downarrow}^\dagger(x, \kappa^\perp) &= -\frac{gT^a}{\sqrt{2(2\pi)^3}} \frac{\sqrt{2}xe^{i\theta}}{|\kappa^\perp|\sqrt{1-x}}, \\
\psi_{\downarrow\uparrow}^\dagger(x, \kappa^\perp) &= \psi_{\downarrow\downarrow}^\dagger(x, \kappa^\perp) = 0.
\end{aligned} \quad (\text{A4})$$

Last two components are zero since we have assumed $m_q = 0$, which forces the quark helicity flip interaction to vanish. Putting them back into eq.(A3) and performing κ^\perp -integration, we get the required result from the two-particle sector. Thus the total contribution from the first term of $|S\rangle$ becomes

$$\begin{aligned}
\Delta q_L^I(1) + \Delta q_L^I(2) &= -\mathcal{N}_q |\Phi_{\text{cm}}(m_1)|^2 \frac{\alpha_s}{2\pi} C_f \ln \frac{\Lambda^2}{\mu^2} (1-x)(1+x) \\
&\quad + \mathcal{N}_q m_1 |\Phi_{\text{cm}}(m_1)|^2 \left\{ \delta(1-x) + \frac{\alpha_s}{2\pi} C_f \ln \frac{\Lambda^2}{\mu^2} \frac{x^3+x}{1-x} \right\}. \quad (\text{A5})
\end{aligned}$$

The contribution from the second term in $|S\rangle$ is again given by eq.(A2) and eq.(A3) with $m_1 \rightarrow m_2$ and $\uparrow \rightarrow \downarrow$. Then one uses

$$\begin{aligned}
\psi_{\uparrow\uparrow}^\downarrow(x, \kappa^\perp) &= \psi_{\uparrow\downarrow}^\downarrow(x, \kappa^\perp) = 0, \\
\psi_{\downarrow\uparrow}^\downarrow(x, \kappa^\perp) &= -\frac{gT^a}{\sqrt{2(2\pi)^3}} \frac{\sqrt{2}xe^{-i\theta}}{|\kappa^\perp|\sqrt{1-x}}, & \psi_{\downarrow\downarrow}^\downarrow(x, \kappa^\perp) &= -\frac{gT^a}{\sqrt{2(2\pi)^3}} \frac{\sqrt{2}e^{i\theta}}{|\kappa^\perp|\sqrt{1-x}}, \quad (\text{A6})
\end{aligned}$$

to obtain

$$\begin{aligned}
\Delta q_L^{II}(1) + \Delta q_L^{II}(2) &= \mathcal{N}_q |\Phi_{\text{cm}}(m_2)|^2 \frac{\alpha_s}{2\pi} C_f \ln \frac{\Lambda^2}{\mu^2} (1-x)(1+x) \\
&\quad + \mathcal{N}_q m_2 |\Phi_{\text{cm}}(m_2)|^2 \left\{ \delta(1-x) + \frac{\alpha_s}{2\pi} C_f \ln \frac{\Lambda^2}{\mu^2} \frac{x^3+x}{1-x} \right\}. \quad (\text{A7})
\end{aligned}$$

Note that eq.(A5) and eq.(A7) are similar except the first term having opposite sign and m_1 is replaced by m_2 . Adding them together, we obtain the final result as given in eq.(13).

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